FATIGUE CURVE APPROXIMATION USING THE DANIELS' SEQUENCE AND MARKOV CHAIN

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The possibility of using the model based on the Daniels' sequence and the Markov chain theory for approximation of S-N fatigue curve of composite material is studied. The model allows to see the connection between static strength distribution parameters and parameter of S-N fatigue curve. Although the model is too simple and does not provide numerical coincidence with experimental fatigue test data, but it can explain existence of fatigue limit and can be used as the nonlinear regression model of S-N fatigue curve with and without the fatigue limit. Using this model we can predict changes in the fatigue curve as a consequence of changes in the static strength parameter. Numerical example is given.

Keywords: strength, fatigue life, Markov chain, Daniels' sequence.

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Introduction. Every year the use of composite materials in aircraft building is increasing. In order to ensure the reliability of flight we should study the phenomenon of fatigue of this material. A lot of papers and books are devoted to this problem (see for example [1]). One of the main quantitative characteristics of this phenomenon is the fatigue curve. There are many approaches to its description. For example, the Weibull' equation is used very often: $S - S_{-1} = C(N + B)^{-\alpha}$, where S_{-1}, C, B, and α are some parameters, S is the stress amplitude and N is the corresponding average number of cycles. Seven equations of the quantile fatigue curve are given in [2]. Parameters of these and similar equations have no connections with the parameters of cumulative distribution function (cdf) of tensile strength of composite material component. Our paper is devoted manly to development of some idea already studied in [3,4]: to find the connection of tensile strength distribution parameters and parameters of fatigue curve, S-N, for unidirectional composite using the model, based on the Markov chain theory, with state space defined by the Daniels' sequence [3]. The successful fitting of experimental fatigue curves can be considered as a proof of "likelihood" of the studied model.

Daniels' sequence model. In Daniels' papers [5, 6], a relationship between the distribution functions of fiber strength and strength of an aggregate of parallel fibers at a uniform distribution of load between them was determined. "Developing" this model in time, we come to a sequence of local stresses $\{s_0, s_1, s_2, ...\}$ which are called the Daniels' sequence (DS) [3]: $s_{i+1} = S/(1 - F(s_i)),\quad i = 1,2,...$, where $s_0 = S$ iis the initial rated stress in the undamaged specimen, $F(s)$ is the cdf of tensile strength. DS can be considered as a sequence of stresses in the cross section, where the failure proceeds, during fatigue loading at the constant mode of loading. It has the following specific feature. If initial stress, S, is over some value (DS-fatigue-limit (DSFLm)), then stress-sequence will grow up to infinity. DSFLm is defined as the maximum value of S for which there is a solution of equation $s = S/(1 - F(s))$. This value is equal to $S_D = \max x(1 - F(x))$. Growth of local stress corresponds to the decrease of local cross section. Let us define that the failure of specimen takes place if local cross section area goes below some value of p_C (initial cross section area is equal to one). Then critical local stress corresponding to this event, S_{UT}^*, is defined from the equation $F_S(S_{UT}^*) = 1 - p_C$. The number $N = k_m \max\{i : s_i < S_{UT}^*\}$, where k_m is some scale coefficient, can be called the Daniels' fatigue life (DSFLf) at the stress S.

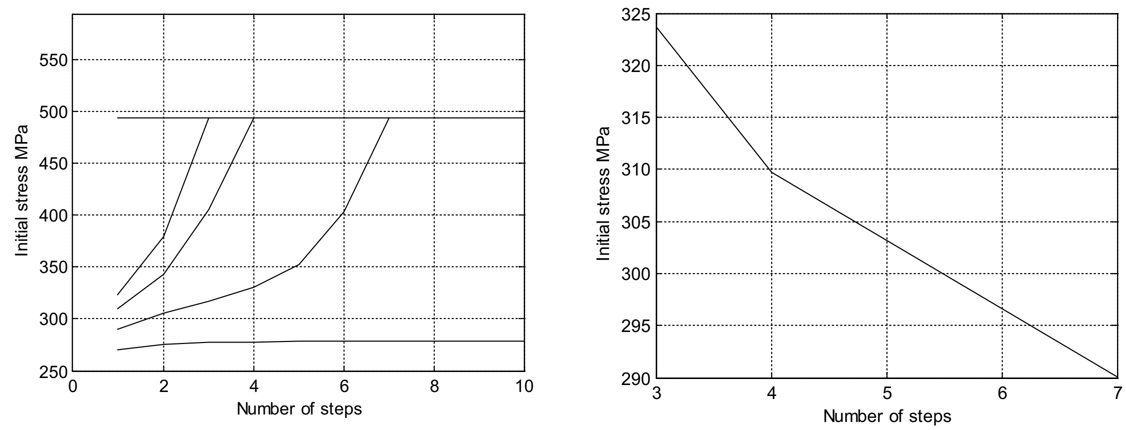

Fig.1. Daniels' sequence of local stresses (a) and the corresponding D-fatigue curve for S =323.7, 309.7 and 290.1 MPa (b) for for k_S =1.6, k_m =1 , p_C = 0.1, S_{UT}^* =494 MPa.

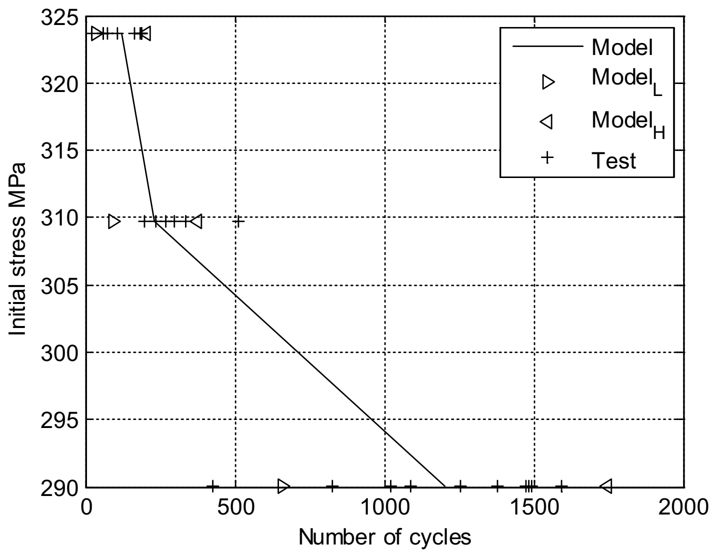

Fig. 2. Fatigue test data (+) and Markov model mean fatigue curve for k_S =1.6 and k_m =12.2847; simbols (► ◄) show two standard deviation intervals.

Here we consider the data of fatigue testing of carbon-fiber composite [7]. In accordance with [7] it was supposed that tensile strength of carbon fiber strands has cdf of lognormal distribution, $F_S(x) = \Phi((\log(x)-\theta_0)/\theta_1)$, where $\Phi(.)$ is cdf of standard normal distribution, with the parameters θ_0 = 6.44 and θ_1 = 0.1816. These carbon fiber strands are longitudinal items of specimens used for the fatigue test. But if we try to calculate DS for the corresponding maximum cycle stresses: (S_1, S_2, S_3) = = (323.7 309.7 290.1) we will see that these stresses are under DSFLm, which is equal in this case to 446.85 MPa. Corresponding DSFLfs are equal to infinity!

So in the framework of the DS-model using the cdf of strength of strands, the failure of specimens can be explained only by existence of significant local stress concentration. Results of calculations of the DSs for the same set of initial stress (S_1, S_2, S_3) taking into account the stress concentration coefficient, k_S =1.6, are given in Fig. 2a. In order to illustrate the explanation by the DS-model of existence of the limit fatigue life phenomenon, the results of calculation for S = 270 MPa are given as well. In the last case the DSFLf is equal to infinity.

In Fig. 1a we see that the DFLf (the order number of DS up to failure of specimen) for k_m = 1 is very small: 3,4,7. So although the DS allows to make quality explanation of fatigue failure of the material, as well as the phenomenon of fatigue limit, but the quantity prediction is very poor. And it does not explain the scatter of fatigue life. But the possibility to explain the phenomenon of fatigue limit is very attractive. So next we will study the model based on the theory of Markov chain with the space of states based on DS.

Simple Markov chain model. We consider to be the Markov chain the one, the first r states of which are related with items of the Daniels' sequence $\{s_0, s_1, ..., s_{r-1}\}$, ($r+1$)-th is the absorbing state (local stress is equal or more than S^*_{UT}) . We assume that the only transitions to the nearest 'senior' states can take place, and we have the following matrix of transition probabilities:

$$P = \begin{bmatrix} q_1 & p_1 & 0 & & \cdots & 0 \\ 0 & q_2 & p_2 & 0 & \cdots & 0 \\ 0 & 0 & q_3 & p_3 & 0 \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & & \cdots & 0 & q_r & p_r \\ 0 & & \cdots & 0 & 0 & 1 \end{bmatrix},$$

$q_i = 1 - p_i$, $i = 1, ..., r$.

The main characteristics of this type of Markov chain are well known. Time to failure (time to absorption) $T = X_1 + X_2 + ... + X_r$, where X_i ((time the process spends in i-th state), i = 1,…,r, are independent random variables. Random variable X_i has geometric distribution with the probability mass function of $P(X_i = n) = (1-p_i)^{n-1} p_i$, $i = 1, 2, ...$ Expectation value and variance are equal to $E(X_i) = 1/p_i$ and $V(X_i) = (1-p_i)/p_i^2$. Probability generating function for random variable T is equal to $G_T(z) = \sum_{i=1}^{\infty} p_T(i) z^i = \prod_{i=1}^{r} \frac{z p_i}{1 - z(1-p_i)}$. The cumulative distribution function of the number of steps up to the specimen failure (number of steps of Markov chain up to absorption in the absorbing state), T_A, is defined by the equation $F_{T_A}(t, S, \eta) = \pi P^t b$, $t = 1, 2, 3, ...$, where the row vector is $\pi = (1, 0, ..., 0)$, the b is column vector (0, … 0,1)'. All these formulas are well known. A new step which we offer to do is the following: 1) the connection of probabilities p_i, $i = 1, ..., r$, with parameter of composite material component strength distribution and parameters of cycles of fatigue loading and 2) the connection of Markov chain state space with the DS.

In what follows, loading by a pulsing cycle is assumed for definiteness; S is the maximum (nominal) stress of the cycle, and η is the vector-parameter (its components are parameters of the distribution functions of strength,...). It is assumed that one step of the Markov chain in general case corresponds to k_M cycles (the k_M is also a component of the vector η). Then fatigue life (the fatigue cycle number up to the specimen failure), T, is equal to $k_m T_A$. The p-quantile fatigue curve which defines the fatigue life $t_p(S)$ (the number of cycles) corresponding to the probability of failure p under an initial normal stress S and the corresponding mean fatigue curve are defined by equations $t_p(S) = k_m F^{-1}_{T_A}(p; S, \eta)$, $E(T(S)) = \int_0^{\infty} t dF_{T_A}(t; S, \eta)$. By fitting the experimental data we can get the estimate of the parameter η (first of all, the values k_m and k_S), by using either the nonlinear method of least squares or the method of maximum likelihood.

In Fig. 2 we see the example of fitting of the data of [7] using the Markov chain model and the same cdf of tensile strength of strands as in the example in the Fig.1, additionally assuming that k_S =1.6 and k_m =12.2847. The items of matrix P are defined in the following way: $p_1 = \Phi((\log(S)-\theta_0)/\theta_1)$; $s_2 = S/(1-p_1)$; $p_{ic} = \Phi((\log(s_i)-\theta_0)/\theta_1)$, $s_{i+1} = S/(1-p_{ic})$, $p_i = (p_{ic} - p_{(i-1)c})/(1-p_{(i-1)c})$, $i = 1,2,...,r$.

Conclusion

The use of the Daniels' sequence and the cdf of strength of longitudinal items (strands) allows to explain the existence of fatigue limit, but its value is too high, and fatigue failure under loading at stress level lower than its value can be explained by the local stress concentration (or local decreasing of strength). But in this case "predicted DS-fatigue life" is too small. Reasonable fitting of fatigue testing data of carbon-fiber composite specimen was obtained using the Markov chain model with states of space based on the Daniels' sequence, taking into account the local stress concentration and a certain scale factor. Although the model is too simple and does not provide too precise numerical coincidence with experimental fatigue test data, it can explain existence of fatigue limit, and it can be used as the nonlinear regression model of S-N fatigue curve with and without the fatigue limit. Using this model we can try to predict changes in the fatigue curve as a consequence of tensile strength parameter changes.

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